Kinematic Relations Between Master and Slave Cylinders in Radial Engines

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Nomenclature

α_o Slave cylinder angle

α_s Link-pin angle

γ_2 Angle between slave cylinder centerline and the line joining the crankshaft journal center with the link pin center

γ_c Angle between crank and slave cylinder centerline

γ_m Master connecting rod angle

γ_s Slave connecting rod angle

θ_c Crank angle

C Clearance between piston and cylinder head

j The cylinder number of a slave cylinder, ranging from 2 to N

l_m Length of the master connecting rod

l_p Piston length

l_s Length of the slave connecting rod

m subscript indicating a parameter for the master cylinder

N The number of cylinders in the engine

r_c Distance between center of crankshaft journal and center of crankshaft

R_h Cylinder head radius

R_p Position of piston top surface

r_s Link-pin radius

R_w Position of wrist pin

s subscript indicating a parameter for the slave cylinder

1 Introduction

Radial engines have a set of cylinders lying in one or more planes, angularly spaced around the crankshaft main bearings. The crankshaft will have one journal for each plane of cylinders. One piston will be connected to the crankshaft journal with a master rod. The remaining pistons will be connected by a slave rod to a link-pin hole in the master rod.

Because the connection to the crankshaft is different for the slave rods than for the master rod, the slave pistons will have different TDC position, stroke, and TDC timing than the master piston. Careful design of the link-pin location, slave rod, and slave cylinders can lead to consistent TDC position for all cylinders, and hence to a consistent compression ratio.

In order to design the desired TDC characteristics, it is desirable to have a kinematic model of the master piston and a slave piston. This paper describes a kinematic model that can be used for radial engine design.

2 Definition of Variables

2.1 Design Variables

Consider a radial engine having N cylinders, spaced at equal angular intervals.By definition, the master cylinder will be called cylinder number 1, and will be located at an angle of zero degrees. Cylinders 2 through N will be slave cylinders. Cylinder j and will be located at an angle α_o, where

$$\alpha_o = \frac{(j-1)*360}{N} \tag{1}$$

The crankshaft journal is located at a radial distance r_c from the center of the crankshaft. The center-to-center distance between the wrist pin hole and the crankshaft journal bearing on the master connecting rod is l_m. The length of the slave connecting rod, l_s, is the center-to-center distance between the wrist pin and the link pin.

The link pin is located on the master connecting rod by two dimensions, and radius and an angle. The link-pin radius r_s is the distance between the center of the crankshaft journal and the center of the link pin. The link-pin angle α_s is the fixed angle between the centerline of the master connecting rod and the line joining the center of the crankshaft journal with the center of the link pin.

The radial distance of the cylinder head from the center of the crankshaft is given by R_h. Note that the master and the slave cylinders can have different cylinder head radii. If they are different, they are identified by an additional subscript of s for slave and m for master. The distance between the wrist pin and the top of the piston is defined as the piston length l_p. Again, the master and slave piston lengths can be different, and will be identified with an additional subscript if necessary.

2.2 Operating Variables

The crank angle θ_c is defined as the angle between the crank journal and the centerline of the master cylinder. The master connecting rod angle γ_m is the angle between the centerline of the master connecting rod and the centerline of the master cylinder. The slave connecting rod angle γ_s is the angle between the centerline of the slave connecting rod and the centerline of the slave cylinder.

The location of the wrist pin along the cylinder centerline is defined to be R_w, and can be defined for both master (m) and slave (s) cylinders. The location of the piston surface is defined to be R_p, and can also be defined for the master and slave cylinders. The clearance between the cylinder head and the piston is C, with an optional subscript indicating the cylinder.

3 Kinematic Model Derivation

The object of the kinematic model is to determine all of the operating variables as a function of the crank angle. When this has been done, we can vary the crank angle from 0 to 360 degrees, and directly measure the TDC position and timing of the slave cylinder, as well as the stroke of the slave cylinder.

Figure 1 shows a kinematic skeleton diagram of the master and slave cylinders. Based on this figure, we can derive expressions for all of the operating variables.

3.1 Master Cylinder

We begin by noting that the vertical position of the crankshaft journal can be calculated independently from the crank angle and the master connecting rod angle. Equating these two expressions, and solving for γ_m, gives the following:

$$r_c \sin\theta_c = l_m \sin\gamma_m \tag{2}$$

$$\gamma_m = \sin^{-1}\left(\frac{r_c \sin\theta_c}{l_m}\right) \tag{3}$$

With γ_m determined, the other operating variables of the master cylinder are easily defined.

$$R_{wm} = r_c \cos\theta_c + l_m \cos\gamma_m \tag{4}$$

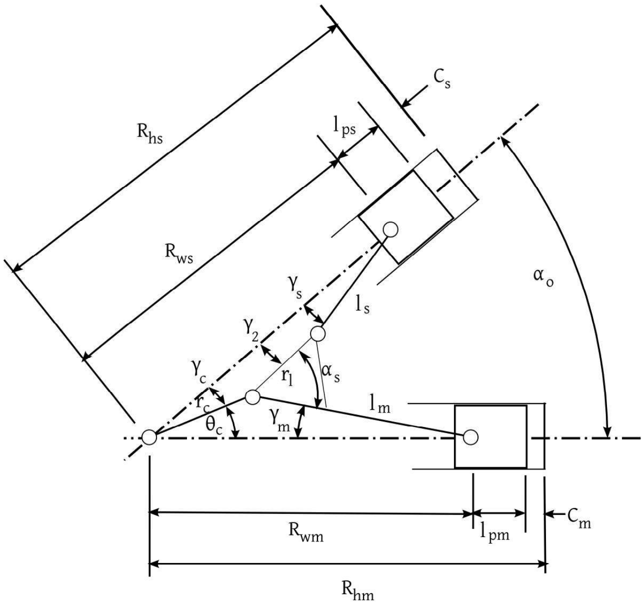

Figure 1: Kinematic skeleton diagram of master and slave cylinders.

$$R_{pm} = R_{wm} + l_{pm} \tag{5}$$

$$C_m = R_{hm} - R_{pm} \tag{6}$$

3.2 Slave Cylinder

To define the operating parameters for the slave cylinder, it is convenient to use the angles the components make with the slave cylinder axis for calculations. We therefore define two new working angles. γ_c is the angle between the crankshaft journal and the slave cylinder centerline, and is given by

$$\gamma_c = \alpha_o - \theta_c \tag{7}$$

γ_2 is the angle between the slave cylinder centerline and the line joining the crankshaft journal center with the link pin center. It is given by

$$\gamma_2 = \alpha_o - \alpha_s + \gamma_m \tag{8}$$

As we did with the master cylinder, we note that there are two independent expressions for the distance of the link-pin from the slave cylinder centerline. We can then equate the two and solve for γ_s.

$$r_c \sin \gamma_c + r_l \sin \gamma_2 = l_s \sin \gamma_s \tag{9}$$

$$\gamma_s = \sin^{-1}\left(\frac{r_c \sin\gamma_c + r_l \sin\gamma_2}{l_s}\right) \tag{10}$$

We can then find the wrist pin location, the piston surface location, and the cylinder head clearance.

$$R_{ws} = r_c \cos\gamma_c + r_l \cos\gamma_2 + l_s \cos\gamma_s \tag{11}$$

$$R_{ps} = R_{ws} + l_{ps} \tag{12}$$

$$C_s = R_{hs} - R_{ps} \tag{13}$$

4 Numerical Calculations

A Microsoft Excel workbook has been developed to numerically calculate the kinematics of a radial engine. For this calculations, dimensions from a 9-cylinder radial engine developed by Garl were used. The first worksheet in the workbook is for the uncompensated engine design, where all slave connecting rods have the same equivalent length as the master connecting rod, i.e.

$$l_s = l_m - r_l \tag{14}$$

and the link pins are equally spaced about the crankshaft journal, i.e.

$$\alpha_s = \alpha_o \tag{15}$$

Further, all pistons have the same dimensions, as do all of the cylinder heads.

To use the worksheet to evaluate a particular slave cylinder, it is only necessary to change the entry in the slave cylinder # cell. All of the angles will automatically adjust for the appropriate cylinder.

As can be seen, in the uncompensated design, the TDC position of the piston is different for the slave cylinder than for the master cylinder. The TDC position, the stroke, and the TDC timing are shown for the all of the cylinders in the uncompensated 9-cylinder engine in Table 1. Note that cylinders that are close to perpendicular to the master cylinder have the largest errors, and the cylinders that are almost directly across from the master cylinder have the smallest errors.

Later worksheets in the workbook show the effects of compensating by changing values of design variables. The optimal solution from a manufacturing standpoint would be to make changes only in the link pin locations, as this would allow full interchangeablilty of pistons, cylinders, and slave connecting rods.

To see what compensation can do, you could use the solver function in excel, and try to set the error in timing, TDC position, or stroke to zero by varying one of the compensation values for the slave cylinder. The solver should be set to the settings tat were used to create the values in the current worksheet.

Table 1: Piston Parameters for Uncompensated 9-cylinder Radial Engine

Cylinder	**TDC Height**	**Stroke**	**TDC Timing**
1	0	1.125	0
2	0.0057	1.1275	37
3	0.0143	1.1275	77
4	0.0116	1.1253	119
5	0.0018	1.125	160
6	0.0018	1.125	200
7	0.0116	1.1253	241
8	0.0143	1.1275	283
9	0.0057	1.1275	323

To date, I have been able to get the TDC position and stroke length errors to be very close to zero, but with variable TDC timing.

I have also been able to get the TDC position and TDC timing to be equal, but with variable stroke.

I have not been able to get TDC position, stroke, and TDC timing errors all to be very close to zero simultaneously.

5 Conclusion

A kinematic model of a master cylinder and slave cylinder for a radial engine has been developed.

The model has been implemented in an excel worksheet.

The worksheet can be used to apply compensation to various design parameters to minimize differences between cylinders.

No set of compensation parameters has been found that is “perfect”, i.e. that eliminates errors in TDC position, stroke, and timing simultaneously.